Extending the Realizable Bandwidth of Edge-Coupled Stripline Filters

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Abstract—**This paper discusses the physical realizability of wide-band parallel-coupled filters in edge-coupled stripline form. Based on the sine-plane approximation, a design procedure that permits the presently realizable bandwidth to be extended from 20 to 45 percent is developed. The new design optimizes the interior impedance level and replaces the end-coupled line sections by quarter-wavelength transformers.**

I. Introduction

Of THE MANY different ways of designing and fabricating microwave bandpass filters, the parallel-coupled filter in printed-circuit form is perhaps the most popular. The circuit is compact, requiring no grounding of resonators. The filter is therefore simple to manufacture and inexpensive to mass produce.

Cohn [1], Matthaei [2], Cristal [3], and Horton and Wenzel [4] have suggested various ways of designing the parallel-coupled filter. In [2], Matthaei stated that filters of up to 20-percent bandwidth can be realized in edge-coupled printed-circuit form. Attempts to widen the bandwidth any further would render the line gaps too narrow for them to be physically realizable. For wide-bandwidth filters, therefore, most designers resort to using other stripline forms that are more amenable to tight coupling.[1] These stripline forms, in general, demand a multilayer construction and, hence, require a careful alignment procedure during assembly. In short, they are more expensive to fabricate.

When the wide-bandwidth filter becomes unrealizable, invariably it is the two end sections that are too tightly coupled. These two end sections are "redundant" elements[2] that function as impedance transformer. The failure, therefore, stems not from the interior filter proper, but from an improper choice of impedance transformer. The question then arises, as to whether a more suitable transformer section exists.

As an impedance transforming device, the parallel-coupled line section is *narrow*-banded, with a *large* equivalent turns ratio (greater than 2.5). Preliminary calculations show that for wide-bandwidth filters (40 percent, say), the interior filter structure requires a *moderate* bandwidth impedance transformer, with a *small* equivalent turns ratio (around 1.7). A quarter-wavelength transformer fulfills these requirements extremely well.[3] For the wide-bandwidth filters therefore, it seems more logical to substitute the quarter-wavelength transformers for the two end sections. The resulting circuit can still be fabricated in edge-coupled stripline form like its narrow-bandwidth brethren. Its cost is thus substantially lower than that of using the offset broadside coupled stripline form favored by most designers [11].

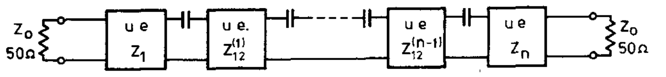

Fig. 1. λ-plane equivalent circuit of the parallel-coupled filter, including the quarter-wavelength transformers. u.e. = unit element, $\lambda = \tanh \tau p$.

II. Quarter-Wavelength Transformer Parallel-Coupled Filter

The previously discussed circuit is depicted in Fig. 1 in its λ-plane equivalent form. The two unit elements designated by Z_1 and Z_n are quarter-wavelength transformers. Although this circuit has the same generic form as the original coupled-line filter, the exact synthesis approach of Horton and Wenzel [4] cannot produce the bandpass filter circuit of Fig. 1 with acceptable line parameters.[4] Under the circumstances, it would be simpler to compromise on an appropriate approximation. An approximate design procedure outlined in Table I is developed as follows.

2.1 *Parallel-Coupled Filter Design by Sine-Plane Approach* [5]

Fig. 2(a) depicts the parallel-coupled filter without the two quarter-wavelength transformers. By the series of transformations shown in Fig. 2(b) and (c), we obtain the sine-plane presentation of its low-pass equivalent circuit [5]. Except for the terminations, which are frequency dependent resistors, the sine-plane circuit is identical to the low-pass prototype circuit of Fig. 3. The low-pass prototype circuit is expressed in Dishal's [6] coupled resonator form, which facilitates the use of predistortion techniques. Its element values must satisfy the following:[5]

$$l_1 = r_1 Q_1$$
$$l_n = r_n Q_n$$
$$z_{i,i+1} = K_{i,i+1}\sqrt{l_i l_{i+1}}, \qquad i = 1, \cdots, n-1. \qquad (1)$$

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[1] See, for example, in Matthaei, Young, and Jones [9, Fig. 13.01-2 (d)–(h), p. 777].

[2] Redundant because they do not increase the selectivity of the filter.

[3] A quarter-wavelength transformer may be regarded as a degenerate coupled-line section with infinite coupling.

[4] Unless one is prepared to go to extreme length, for the example in Section III, the simple synthesis procedure such as in [4] would yield either $Z_1 = 50\ \Omega$ (if the u.e.'s are redundant) or $Z_1 \simeq 200\ \Omega$ (if the u.e.'s are nonredundant). Neither of the two solutions can produce a stripline structure which is physically realizable in edge-coupled form.

[5] Lowercase letters are reserved for impedance normalized quantities. Uppercase letters for unnormalized quantities.

TABLE I
DESIGN OF PARALLEL COUPLED FILTER WITH QUARTER-WAVELENGTH TRANSFORMERS

1) *Nomenclature*

a) The filter consists of $(n-1)$ coupled line section and 2 quarter-wavelength transformers at both ends.
b) w: Fractional bandwidth.
c) q_i: Normalized Q of the resonators in the prototype network. $k_{i,i+1}$: Normalized coupling coefficients of the prototype network. Both q_i and $k_{i,i+1}$ available from Zverev [7].
d) r_i: The normalized interior filter terminating impedance level (see Fig. 2(a)).
e) $$s_0 = \sin\left(\frac{\pi}{4}w\right), \qquad c_0 = \cos\left(\frac{\pi}{4}w\right).$$
f) Physical dimensions: W = strip-width; S = gap width; B = Ground plane spacing.
g) Z_0: System characteristic impedance (usually 50 Ω).

2) *Denormalized q's and k's with Respect to Bandwidth:*

$$Q_i = q_i/s_0, \qquad i = 1,n$$
$$K_{i,i+1} = k_{i,i+1}s_0, \qquad i = 1, 2, \cdots, n-1.$$

3) *Recommended Filter Terminating Impedance Level r (with Respect to System Impedance of Z_0):*

$$r_i = \frac{110}{\sqrt{\varepsilon_r}\, Z_0 Q_i}, \qquad i = 1,n,$$

for edge-coupled stripline system (to ensure $W/B \gtrsim 0.35$) where ε_r is the relative dielectric constant of substrate.

4) *Recommended Values for l_i:*

$$l_1 = r_1 Q_1 = \frac{110}{\sqrt{\varepsilon_r}\, Z_0}$$
$$l_n = r_n Q_n = l_1$$
$$l_i = 2l_1, \qquad i = 2, 3, \cdots, n-1.$$

5) *Normalized Line Parameters (with Respect to Z_0)*

Self Impedance:

$$z_{22}^{(i)} + z_{11}^{(i+1)} = l_{i+1}, \qquad i = 0, 1, \cdots, n-1$$

where

$$z_{22}^{(0)} = z_{11}^{(n)} = 0.$$

$z_{11}^{(i)} = z_{22}^{(i)}$ is recommended to give symmetrical sections in which case

$$z_{11}^{(i)} = z_{22}^{(i)} = l_1 = \frac{110}{\sqrt{\varepsilon_r}\, Z_0}, \qquad i = 1, 2, \cdots, n-1.$$

Transfer Impedance:

$$z_{12}^{(i)} = K_{i,i+1}\sqrt{l_i l_{i+1}} = \frac{220}{\sqrt{\varepsilon_r}\, Z_0} K_{i,i+1}, \qquad i = 2, 3, \cdots, n-2.$$

6) *Determine y_i*

y_i is the positive real solution of

$$y_i^3 + \frac{1}{l_1}y_i^2 - \left(\frac{1}{r_i c_0} + 1 - \frac{1}{c_0^2}\right)y_i - \frac{1}{l_1} = 0, \qquad i = 1,n.$$

y_i may be determined iteratively commencing with $1/\sqrt{r_i}$ as a first approximation or explicitly as follows. Let

$$a = \frac{1}{3l_1}, \qquad b = \frac{1}{r_i c_0} + 1 - \frac{1}{c_0^2}$$
$$D = \sqrt{a^2 + b/3}, \qquad E = \tfrac{1}{2}a(3 - 2a^2 - b).$$

Then

$$y_i = 2D\cos\left[\tfrac{1}{3}\cos^{-1}\left(\frac{E}{D^3}\right)\right] - a.$$

7) *Compute the Transfer Impedance of the 1st and $(n-1)$th Section:*

$$l_1 = l_1 + (y_1 - 1/y_1)/(y_1^2 + s_0^2/c_0^2)$$
$$l_{n+1} = l_n + (y_n - 1/y_n)/(y_n^2 + s_0^2/c_0^2)$$
$$z_{12}^{(1)} = K_{12}\sqrt{l_1 l_2}$$
$$z_{12}^{(n-1)} = K_{n-1,n}\sqrt{l_{n-1} l_{n+1}}.$$

8) *Denormalized All the Line Impedance Parameters:*

$$Z_{ij}^{(k)} = Z_0 \cdot z_{ij}^{(k)}, \qquad k = 1, \cdots, n-1, \quad i = 1,2, \quad j = 1,2$$
$$Z_i = Z_0/y_i, \qquad i = 1,n.$$

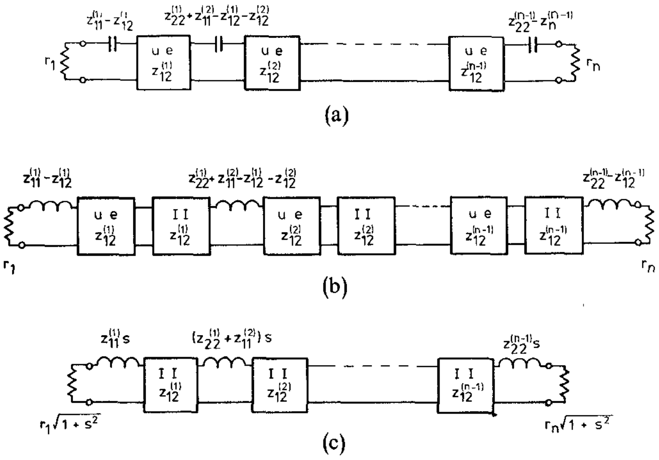

Fig. 2. Parallel-coupled filter and its sine-plane presentation. (a) λ-plane presentation of the parallel-coupled filter, without the quarter-wavelength transformers. (b) λ-plane low-pass equivalent of (a) above. I.I. = impedance inverter. (c) Sine-plane presentation of the low-pass equivalent circuit in (b) above. $s = \sinh \tau p$.

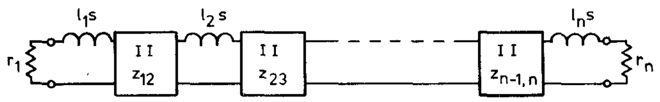

Fig. 3. Low-pass prototype filter. Impedance scaled by r_1 and frequency scaled by s_0.

The parameters in (1) are defined in steps 1) and 2) of Table I. The q and k values are readily available from filter design handbooks for any standard filter characteristic [7].

By equating the element values of the sine-plane low-pass equivalent circuit of Fig. 2(c) to the corresponding values in the prototype circuit of Fig. 3, we obtain the set of design formula listed in step 5) of Table I. Note that the choice of l_i in step 4) ensures that z_{11} of all coupled line sections are identical.

2.2 *Choice of Normalized Filter Impedance Level r*

Although filter impedance level r_i may be set at an arbitrary level, we recommend that r_i (usually < 1) be set at a *high* impedance level for two reasons. First, the step size of the quarter-wavelength transformer would be smaller as a result. Second, the gap sizes of the coupled line sections would be wider, thereby reducing the tight tolerance problem in the filter interior. The value recommended in step 3)

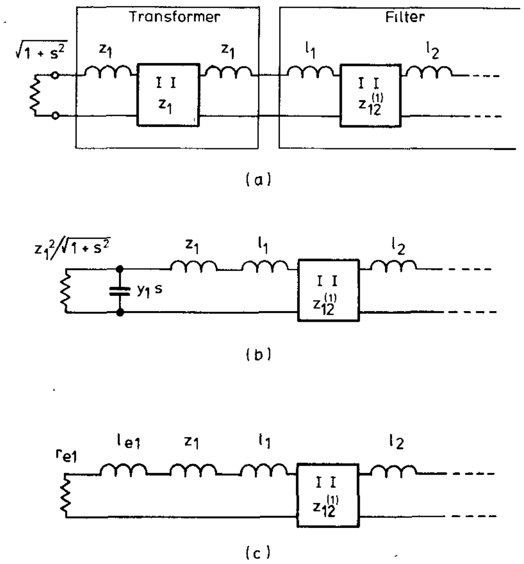

Fig. 4. Loading of the filter by the quarter-wavelength transformer in the sine-plane equivalent circuit; (a) and (b) are exact, (c) is the band edge equivalent of (b).

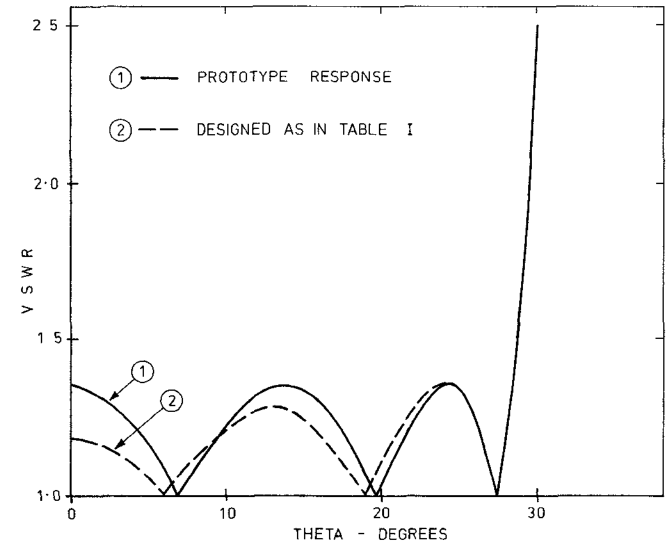

Fig. 5. Passband VSWR response for 70-percent bandwidth, 0.1-dB ripple, 6 resonator Chebyshev filter and its sine-plane approximation.

of Table I is the highest possible value without violating the narrow line constraint set in Getsinger [8]. This value can be lowered for the offset broadside coupled form of construction. It is mainly in the choice of interior impedance level that the interior coupled sections of the new design differ from those of previous designs. However, this choice necessitates more than a mere impedance scaling of the interior sections, as computed by previous designs.

2.3 *Quarter-Wavelength Transformer*

Once the filter impedance level r_i has been chosen, the normalized transformer impedance z_i may be obtained from

$$z_i = \sqrt{r_i}, \qquad i = 1,n. \tag{2}$$

However, (2) is only a first-order approximation. By including the quarter-wavelength transformers into the circuit, the "excess" transformer reactance would modify the first and last resonators by a significant amount. To correct for this effect, a second-order approximation is developed as follows.

Fig. 4(a) shows the first coupled line section loaded by the source (1 Ω) through a quarter-wavelength transformer, where the circuit in the enclosed box represents the low-pass sine-plane equivalent circuit of the transformer. The transformer thus presents an *RLC* circuit to the filter as shown in Fig. 4(b). At the band edge frequency ($s = js_0$), the parallel *RC* section can be converted into an equivalent *RL* section (negative *L*) as shown in Fig. 4(c), where

$$\begin{aligned} r_{e1} &= c_0/(c_0^2 y_1^2 + s_0^2) \\ l_{e1} &= -z_1/(c_0^2 y_1^2 + s_0^2) \end{aligned} \tag{3}$$

and

$$y_1 = 1/z_1.$$

The first resonator shown in Fig. 4(c) now has a total inductance ($l_{e1} + z_1 + l_1$). Consequently the first (and last) coupled line section must be modified accordingly. Similar relationships as (1), viz.,

$$\begin{aligned} l_0 &= l_{e1} + z_1 + l_1 \\ z_{12}^{(1)} &= K_{12}\sqrt{l_0 l_2} \\ l_0 &= Q_1 r_{e1} \end{aligned} \tag{4}$$

must again be maintained to preserve the same frequency characteristic at the band edge frequency. Equations (3) and (4) lead to a cubic equation for y_1. The equation has one positive real root that may be determined iteratively or explicitly as described in step 6) of Table I. Step 7) of Table I follows from (4).

2.4 *Amplitude Response of the Composite Filter*

The amplitude characteristic of a 70-percent bandwidth, 0.1-dB passband ripple, 6 resonator Chebyshev low-pass filter is shown in Fig. 5 (curve 1 and 2, respectively). Except for a reduced VSWR at the band center,[6] the sine-plane approximation matches the prototype response very well.

The comparison thus shows that the sine-plane approximation is satisfactory to at least a 70-percent bandwidth.

III. Worked Example

This section illustrates the use of the design in Table I by designing a 40-percent bandwidth, 6 resonator, 0.1-dB equiripple filter. Assume $\varepsilon_r = 2.53$, $B = \frac{1}{4}$ in (6.35 mm), $Z_0 = 50$ Ω.

1) Prototype Filter Parameters (Steps 1) and 2))

$$w = 0.4$$

$$s_0 = 0.3090$$

$$c_0 = 0.9511.$$

[6] The reduced VSWR at the band center can be explained in the same way as in Cristal [3].

From Zverev [7, p. 348] (for lossless case $q_0 = \infty$)

$$q_1 = q_6 = 1.2767$$
$$k_{12} = k_{56} = 0.7145$$
$$k_{23} = k_{45} = 0.5385$$
$$k_{34} = 0.5180.$$

Denormalize k's and q's with respect to s_0

$$Q_1 = Q_6 = 4.1315$$
$$K_{12} = K_{56} = 0.2208$$
$$K_{23} = K_{45} = 0.1664$$
$$K_{34} = 0.1601.$$

2) Parallel Coupled Filter Parameters (Steps 3), 4), 5))

$$r_1 = r_6 = 0.3348$$
$$z_{11}^{(1)} = z_{11}^{(2)} = \cdots = z_{11}^{(5)} = l_1 = 1.3831$$
$$z_{12}^{(2)} = z_{12}^{(4)} = 0.4603$$
$$z_{12}^{(3)} = 0.4428.$$

3) Determine y_0 of Transforming Sections (Step 6))

$$y_1 = y_6 = 1.5447.$$

4) Adjust End-Coupling Sections (Step 7))

$$l_0 = l_7 = 1.7432$$
$$z_{12}^{(1)} = z_{12}^{(5)} = 0.4848.$$

5) Denormalize (Step 8))

$$Z_{11}^{(1)} = \cdots = Z_{11}^{(5)} = 69.16\ \Omega$$
$$Z_{12}^{(1)} = Z_{12}^{(5)} = 24.24\ \Omega$$
$$Z_{12}^{(2)} = Z_{12}^{(4)} = 23.02\ \Omega$$
$$Z_{12}^{(3)} = 22.14\ \Omega$$
$$Z_1 = Z_6 = 32.37\ \Omega.$$

6) Determine Physical Dimensions

Using Cohn [10], the dimensions of Table II are obtained.

IV. Experimental Results

The filter designed in Section III was constructed using Rexolite polystyrene dielectric. The center frequency was selected to be 1.6 GHz. The completed filter with top dielectric layer removed is shown in Fig. 6. Note that the resonator lengths were shortened by 0.6 mm to correct for fringing. Fig. 7 shows the measured and computed frequency response. The computed stopband response is slightly steeper than the measured response and the computed passband ripple (0.1 dB) is less than the measured ripple (1 dB).[7] With these exceptions the measured response conforms fairly well with the computed response.

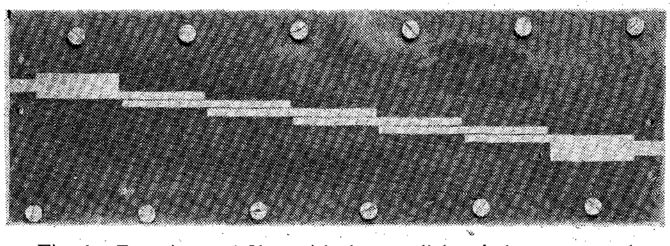

Fig. 6. Experimental filter with the top dielectric layer removed.

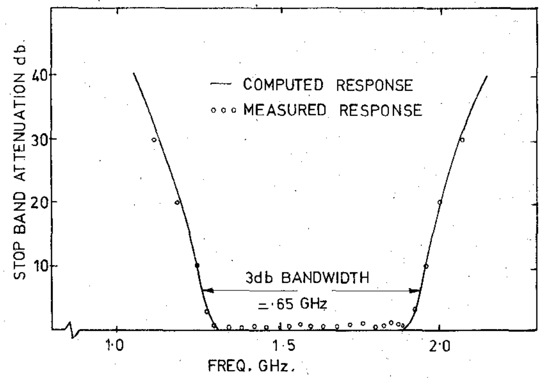

Fig. 7. Frequency response of the experimental filter, based on the new design equations.

TABLE II
Filter Dimensions for Worked Example

SECTION	S/B	S mm.	W/B	W.mm.
50 ohm line	-	-	0.744	4.72
quarter wave transformer	-	-	1.388	8.81
1, 5	0.0691	0.439	0.381	2.42
2, 4	0.0796	0.505	0.385	2.44
3	0.0878	0.558	0.388	2.46

V. Conclusion

A new design procedure, based on the sine-plane approximation to the lumped low-pass prototype filter, was developed for wide-band parallel-coupled filters. The new design is suitable for filters constructed in edge-coupled stripline form in the 20- to 45-percent range of bandwidths. The degree of approximation to the prototype response is about equal to that provided by the previous design of Cristal [3].

Above a 45-percent bandwidth, the interior coupled sections, like the end sections discussed before, become too tightly coupled. In this case, offset broadside coupled stripline form can be used to extend the bandwidth to about 70 percent, at which point other geometrical considerations

[7] Mainly due to tolerance problems in the manufacturing process.

begin to restrict the design. At this bandwidth, however, other filter configurations, such as the shunt-stub filter, may be a better choice.

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A Statistical Measure for the Stability of Solid State Noise Sources

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Abstract—**A statistical measure for the stability of solid state noise sources is discussed. Its applicability is demonstrated. Two independent techniques, a cross correlation technique and a triangulation technique, are employed to estimate the instabilities of individual noise sources. An analysis for the validity of the cross correlation technique for separating the instability of the noise source from that of the measurement system is also given.**

I. Introduction

SOLID STATE noise sources are influenced by their environment, and an unsuitable environment can cause unstable operation. Observed instabilities can be traced to changes in ambient temperature, operating power supply voltages and currents, humidity, etc. In order to unravel the causes of these instabilities, the need for the measurement of the stability of solid state noise sources has received considerable attention [1].

The purpose of this paper is to describe a measure of stability applicable to solid state noise sources. Conventionally, the stability of devices such as amplifiers, signal generators, noise sources, etc., has been given in terms of the degree of changes or fluctuations in the observables for a specific sampling time interval. However, if one wishes to forecast future stabilities of a time series from present and past stability measurements, such a conventional measure for stability is obviously not sufficient, and a new statistical measure for stability is needed. For this purpose, Allan variance analysis, discussed in Section II, is used as a meaningful statistical measure for the stability of solid state noise sources.

Unless an experiment is carefully planned, the instabilities of the measurement system will often obscure those of the noise source under test, and a method for separating the instabilities is required. A technique similar to cross correlation is employed for this purpose and is described in Section 3.1. If a measurement process is used where the instability of the measurement system does not affect the measurement of the stability of the noise source under test, the triangulation technique, described in Section 3.2, will provide an estimate for the instability of noise from an individual noise source. These two independent techniques provide for a check on the validity of the individual measurement schemes used.

A comparison between the stability of the output noise power from commercial solid state noise sources and that from typical commercial argon gas noise sources is given in Section IV. The stabilities at several frequencies between 1 GHz and 18 GHz were examined. Two different manufacturers' sources and two models from each manufacturer that cover the frequency range are used. The noise diodes used for these commercial solid state noise sources are Read-type silicon avalanche diodes. The microwave noise generated by them is expected to be white and Gaussian. The long-term instability measurements over months and years have not been completed; however, the results shown in Section IV will give the statistical expectation for fluctuations of noise

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